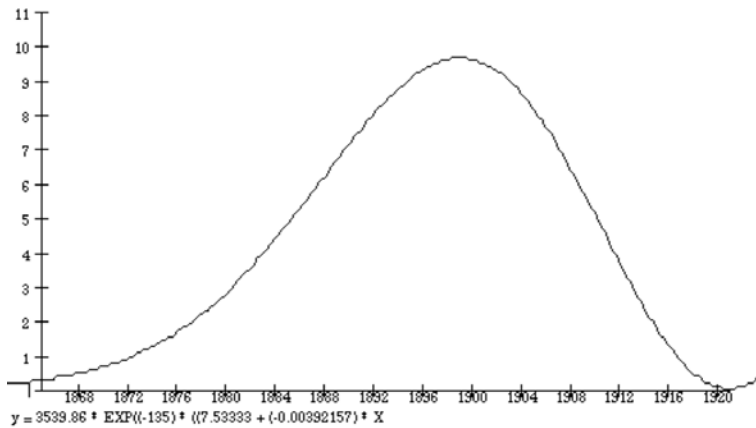


PHE CASE STUDY #1: Colorado San Juans Mining Region Modeled Via Maxwell-Boltzmann Distribution

By Mark P. A. Ciotola



© 1997, 2009 by Mark P. A. Ciotola.

Either portions or all of this work are registered with the U.S. government
Library of Congress.

and covered internationally by the Berne Convention.

Published by Fast Entropy Press

Pavilion of Research and Commerce

210 Fell Street, San Francisco, California 94102

www.fastentropy.com • www.fastentropy.org

All rights reserved.

Supplement One to Physical History and Economics, Second Edition

Preface

Colorado San Juans Mining Region Via Maxwell-Boltzmann Distribution is the first in a case studies series related to material found in *Physical History and Economics, 2nd Edition*. Most case studies will material to that found in *Physical History and Economics, 2nd Edition*, but this particular case was included in full in that book.

This case study was published by the author in the *Journal of Physical History & Economics* in Spring 1997. That issue was self-published and not peer-reviewed. Minor editing has taken place.

This particular case study is more mathematical than some of those that follow, while at the same time the science of Physical History and Economics has moved beyond that contained in this case study. The mathematics contained here represent an intermediate attempt at modeling a regime using physics rather than purely empirical mathematics. Nevertheless, this history is interesting to read, and this is a good example of the general regime pattern involving a conserved (fixed quantity) resource.

The reader is also recommend to obtain and read *Physical History (2nd Edition)*, referred to as the “main book”. Nevertheless, this case study may contain stand-alone material of interest. The non-mathematical portions of this case study are a good introduction to the main book.

Finally, the author reminds the reader that *Physical History and Economics* is still in its early stages, lacks the refinement and rigor the reader may be accustomed to in other works. In this regard, the author is similar to a grizzly prospector in the pioneering era of a new mining region such as the San Juans once were in Colorado. The refinements may come later with the eventual arrival of more “civilized” authors in the field.

Acknowledgements

Those who were thanked under Acknowledgements in the main book are thanked again, especially the reviewers.

Table of Contents

Case Study

Introduction 5

Brief Chronological History 5

Development and use of Maxwell-Boltzmann distribution
function in physics 6

Distribution's use in modeling economic behavior using San
Juans data as an example 6

Generic Mining Town History 9

Conclusion 10

References

C-1

Colorado San Juans Mining Region Modeled Via Maxwell-Boltzmann Distribution

San Juans Mining Region Case Study: Application of Maxwell-Boltzmann Distribution Function

Abstract: This study utilizes an equation derived from the Maxwell-Boltzmann distribution function to model economic output over time for mining production of the San Juans region in Colorado during the 19th and early 20th centuries.

Introduction:

This study will briefly present the history of the San Juans mining region, discuss the development and use of the Boltzmann-Maxwell distribution function in physics and then discuss the distributions use in modeling economic behavior behavior using San Juans data as an example.

Brief Chronological History

(Note that dollar amounts are unadjusted for inflation — they reflect actual historical figures).

The San Juans mining region of Colorado produced gold and silver from dozens of mines, around which towns and communities eventually developed. Mining began as early as 1765. Its heydays were between about 1889 and 1900.¹ There is again mining in miscellaneous minerals, but the not much in gold, which was the primary economic driver for the “great days”. The region is now used primarily for recreation and some agriculture.²

Spanish gold mining of placer deposits took place between about 1765-1776 (native pieces of nearly pure gold found on the surface). Some mining took place in 1860, but it was interrupted by U.S. Civil War. At this point, “only the smaller deposits of high-grade ore could

be mined profitably.” Mining slowly started again in 1869.³ There were 200 miners by 1870.⁴ An Indian Treaty was negotiated in 1873, which removed a major obstacle to an increase of mining.⁵

By 1880 there was nationally a ‘surplus of silver; pressures to lower wages; labor troubles.’⁶ In 1881 a railroad service was established, resulting in a ‘decline in ore shipping rates.’⁷ By 1889, \$1 million in gold and silver were being produced each year (for one particular subregion). Around 1889, English investors had come to control the major mines by this time.⁸ The 1890 production total for San Juans was \$1,120,000 in gold; \$5,176,000 in silver. The region produced saw \$4,325,000 in gold and \$5,377,000 in silver in 1899.⁹

By 1900, the region began to take on more of the characteristics of a settled community. There was a movement for more “God” and less “red lights.”¹⁰ By 1909, “the gilt had eroded” (dilapidation set in; decreasing population).¹¹ In 1914, production greatly fell, due to decreased demand from Europe (because of World War I) and the region lost workers. Farming becomes more important to local economy than mining.¹² Recreation and tourism revenues become the only bright spot for many mining towns.¹³ Silver and gold mining all but ceased by about 1921.¹⁴

Development and use of the Maxwell- Boltzmann distribution function in physics

An equation derived from the Maxwell-Boltzmann distribution function is used to model economic output over time for mining production of this region.

The Maxwell-Boltzmann distribution function is used in physics to describe the distribution of molecular speeds in a gas.

It can be expressed in the form:

$$(1) \quad f(v) = [4 / \pi^{1/2}] [m/2KT]^{3/2} (v^2) [e^{(-mv^2/2kT)}]$$

where m is the mass of an individual molecule of the gas,
k is the Boltzmann constant
and T is the temperature expressed in degrees Kelvin (absolute degrees)

The distribution’s use in modeling economic behavior using San Juans data as an example

Background

It has long been suggested that statistical tools should be used to study human behavior. Science fiction author Isaac Asimov suggested this in his *Foundation* series. Of course statistical analysis is the mainstay of market researchers, political polsters and social science faculty and graduate students.

In particular, the bell-shaped curve of the normal distribution has been used as an alleged crystal ball to explain why one student should get such a grade as compared to another student and the intelligence of different groups as compared with others.

This bell-shaped curve has been of important use in geology, at least as it pertains to economics. M. King Hubbert used a bell-shaped curve to model domestic U.S. petroleum production over time. So far, time has proved him correct.

Conceptually, Hubbert’s model essentially utilizes a normal probability distribution, which is conceptually reasonable, since distribution of national petroleum reserves are widely scattered. However, Hubbert’s model is symmetric around the point of maximum production. Yet, many instances in society and economics are not symmetric over time.

Conceptual Application of Maxwell-Boltzmann Distribution

The San Juans offer such a case. While data obtained this far is extremely limited, there is enough qualitative information to provide a few data points and qualitative information to paint a picture of life in the San Juans. This information primarily draws from the book *Song and Hammer*.

In the case of a mining region, there is a veritable “cloud” of people.

Quantitative Analysis of Application of Maxwell-Boltzmann Distribution

An equation derived from the Maxwell-Boltzmann distribution function is used to model economic output over time for mining production of this region.

As mentioned, supra, the Maxwell-Boltzmann distribution, used in physics to describe the distribution of molecular speeds in a gas, can be expressed in the form:

$$(2) \quad f(v) = [4 / \pi^{1/2}] [m/2kT]^{3/2} (v^2) [e^{(-mv^2/2kT)}]$$

where m is the mass of an individual molecule of the gas,
k is the Boltzmann constant
and T is the temperature expressed in degrees Kelvin (absolute degrees)

It is useful to note that the quantity $m/2kT$ is itself composed of values that together can be considered as a single constant, which we will call “K.” Although the temperature of a real gas of course can vary over time in nature, we can nevertheless consider a distribution for which there is no significant change in temperature. Using our constant K, we have:

$$(3) \quad f(v) = [4 / \pi^{1/2}] [K]^{3/2} (v^2) [e^{(-Kv^2)}]$$

Further note, that of this distribution is plotted from the domain $-\infty$ to $+\infty$ that there is are two maxima, resulting in a “double-humped” curve, instead of the usual single maximum shown when this distribution is traditionally plotted from 0 to $+\infty$. There is symmetry about the y axis. For purposes of this example, we will use the maximum produced when plotting from $-\infty$ to 0, for no better reasons than that there is a better data fit. Specifically, the domain produces a curve with a “tail” for lower values. It may be useful to further study the reasons for this better fit, but the thing to remember is that the total area under the curve is the same as in the traditional domain used.

Further, the distribution in this case study is plotted in unit of monetary value of production versus time to better study its fit with historical data, so units of both variable and constants need

to be adapted. Further, a new constant D is added, particularly for the purpose of producing a function that can be plotted on an axis with units of millions of dollars (old ones).

Using our constant D, we have:

$$(4) \quad f(t) = D [4 / \pi^{1/2}] [K]^{3/2} (t^2) [e^{-Kt^2}]$$

The real exercise is finding a K that will best fit the very limited data. Since there is more of a physical basis (using the Maxwell-Boltzmann analogy) for working with K rather than D, most of the fitting will utilize varying K. D will mostly work with units.

Further, one cannot expect to merely plug in a calendar year into a formula. Dates must be converted into an appropriate time scale. Since the original distribution considers velocity, and we want to consider time, we replace v with t (with awareness of the impact on K), with t being a function of calendar year.

The three specific data points we have are:

Year	Total Production (\$)
1889	~6,000,000
1899	~10,000,000
1921	~ 0

However, the distribution function must also be consistent with anecdotal data, which indicates that there was significant mining activity in the early 1870's as well as some before the U.S. Civil War. It may be very well that there was in reality more mining going on after 1921 than before about 1877, but that the earlier mining has been more "played up" being the stuff of folklore and legends and the later mining being down-played. This would allow us to use the traditional distribution with the tail to the right. However, this is merely speculation and mining has certainly lost its position as the primary economic driver that it has during the initial settlement of the San Juans. The sketchy information obtained thus far about the earlier mining is partially quantitative (i.e. number of miners and productivity of some individual mines and towns) and does indicate activity which must be considered.

The peak economic production chosen for the mining of the region activity is 1899. This is the year which the data obtained indicates the highest level of activity. Even anecdotal indications suggest that the actual peak could not have happened more than a few years later.

With blind trial and error fitting of the constant K and date conversion, we have:

$$(5) \quad f(t) = D [4 / \pi^{1/2}] [135]^{3/2} (t^2) [e^{-135t^2}]$$

using values of: $D = y^{-1/2} \times 1$ million dollars US (old), $K = 135 y^{-1}$ and $t = (1921 - \text{year})/255$.

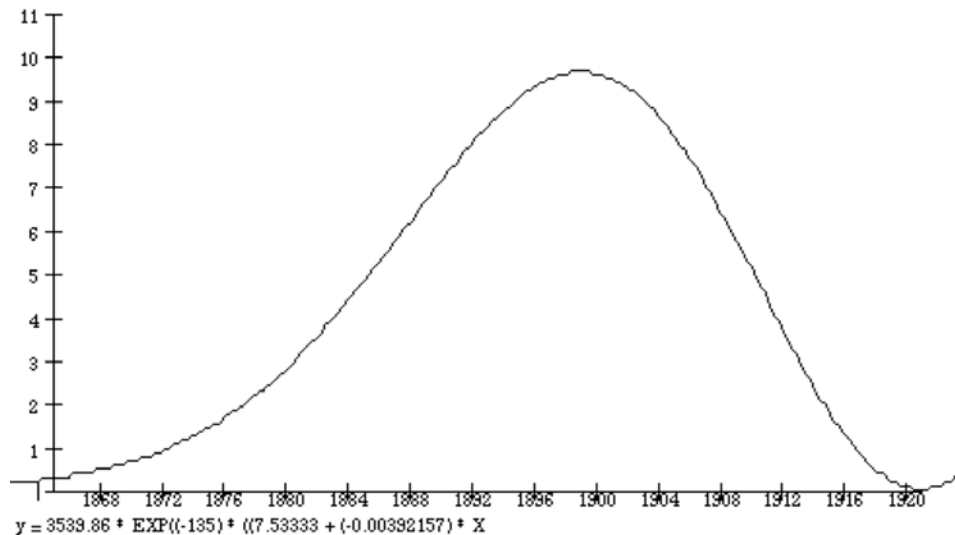


FIGURE C1-1 Hubbert Curve for San Juan region modeled as a Maxwell-Boltzmann distribution (nominal y scale)

Generic Mining Town History

The operation of this distribution can be qualitatively utilized to produce a generic history or profile for a mining region.¹⁵ While the actual economic production of the regions mines will be affected by external events such as national recessions or even wars, the generic profile can be used as a starting point to understand the operation of these external shocks upon the mining region’s actual history. Here is such a generic history

The existence of valuable, high-grade ore is found at a site. Often the site is in the wilderness with no major settlements nearby, since the site is often rocky and not suited to agriculture. Unless the ore is placer (nearly pure gold), the discovery will not yet attract much attention.

Sustained finding of valuable, high-grade ore, often by individual miners who are literally scratching at the surface of the ground, attract serious investigation by larger mining operations.

Small mining camps are established. They tend to be taking advantage of select, high-grade ore veins and despite their inefficiencies, many make a healthy return. Such camps tend to involve the labor on a particular mine, an owner or manager, and perhaps a camp cook, carpenter and a blacksmith.

New veins are often being discovered, and the area gets a reputation for the possibility of making one rich. Many miners and mining operations come to establish stakes. Since most, but not all, of the “easy” areas to mine have already been claimed, newer stakes tend to require more financial capital to try to exploit. Some veins are rich and make their owners a fortune, which enhances the reputation of the area. Many more discoveries turn out to be small, and many fortunes are lost.

A lively mining town begins to develop, including with professions that do not exist purely for the physical operation of mines. There may be a livery facility, sellers of mining equipment, an assayer to buy the findings of the smaller miners. Entertainment exists for the miners.

Since most of the discovered ore near the surface has been removed by now, costs to extract the remaining ore become higher. The cost of extracting the ore moves closer to the market price for such ore. As much of the ore becomes lower grade, transportation costs become more of an issue. While some surface discoveries are still made, some of them very valuable and high grade (which will still attract the fortune seekers), market fluctuations in the price for ore and its underlying valuable metal drive some mines to financial crisis. Larger mining outfits, owned by outsiders who are more often professional financiers than actual miners or technical types, purchase many of the smaller mines. With their capital and financial credibility, such financiers are able to undertake substantial capital improvements that increase the efficiencies of mining such as larger tunnels, better extraction equipment and railroads.

Along with institutionalization of many of the mines, a professional class begins to come to mining town. Hotels, rather than saloons with lodging, need to be built for visiting financiers and professionals. Banks replace the assayers. Better mercantile stores develop to serve the clerical and professional class. Also, miners who have been in the area start to form families. Schools are built to serve the population.

As time passes, and more ore is removed, the costs of extracting much of the remaining ore exceeds the market price for such ore, even though significant amounts of ore may remain in the ground. Indeed, there will be new discoveries of ore that is inexpensive (i.e. near the surface or close to old veins) to exploit, so that mining activities may continue for a long time to come, and there may even be newly-made millionaires, so to speak. Nevertheless, as time passes, an increasing percentage of the remaining ore reserves will become uneconomical to exploit, until mining activity essentially vanishes at the site.

Conclusion

The ability to express economic output of a mining region quantitatively using the Maxwell-Boltzmann distribution is conceptually satisfying, even if the data is limited and the unit transformations not fully substantiated, because the analogy itself does seem appropriate. Throngs of grizzly, independent prospectors scattered widely over a confined region does not seem so unlike the cloud of kinetic, widely-space particles comprising an ideal gas. Even well financed, corporate mining operations (in the later stages of the region's mining life) trying to find mysteriously hidden and placed pockets of gold and silver among mountains of rock seems to fit within the gas cloud analogy.

However, it will be the ability find patterns in the value of K across mining regions and the application of that value of K to other social and economic regimes that will allow development of a truly useful tool for the analysis of history.

REFERENCES

R

References

References

1. E.g., DUANE A. SMITH, SONG OF THE DRILL AND HAMMER: THE COLORADO SAN JUANS, 1860-1914. (1982) (Colorado School of Mines Press) [hereinafter cited as SMITH, SONG OF THE DRILL AND HAMMER].
2. E. g., DUANE A. SMITH, COLORADO MINING 151. (1977) (University of New Mexico Press).
3. SMITH, SONG OF THE DRILL AND HAMMER, *supra*, at 5-9.
4. *Id.*, at 10.
5. *Id.*, at 11-14.
6. *Id.*, at 59.
7. *Id.*, at 50.
8. *Id.*, at 53.
9. *Id.*, at 91.
10. *Id.*, at 153-157.
11. *Id.*, at 159.
12. *Id.*, at 161, 165, 174.
13. See *Id.*, at 161.
14. See *Id.*, at 170.
15. See COLORADO STATE BUSINESS DIRECTORY (issues during the end of the 19th Century and up to about 1920).